Short Distance Repulsive Gravity

as a Consequence of Non Trivial

PPN Parameters β and γ

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ABSTRACT.

We look for a graviton-dilaton theory which can predict non trivial values of the PPN parameters β and/or γ for a charge neutral point star, without any naked singularity. With the potential for dilaton ϕ set to zero, it contains one arbitrary function $\psi(\phi)$. Our requirements impose certain constraints on ψ, which lead to the following generic and model independent novel results: For a charge neutral point star, the gravitational force becomes repulsive at distances of the order of, but greater than, the Schwarzschild radius r_0. There is also no horizon for $r > r_0$. These results suggest that black holes are unlikely to form in a stellar collapse in this theory.

In Einstein's theory of general relativity, the gravitational field of a charge neutral point star is described by static spherically symmetric Schwarzschild solution. Two of the parametrised post Newtonian (PPN) parameters β and γ can be calculated from such a solution. The parameter β is a measure of non linearity in the superposition law for gravity, and γ is a measure of the space time curvature [1]. In Einstein's theory $\beta = \gamma = 1$. Experimentally, they can be measured by perihelion shifts of planets and Shapiro time delay, and are given by $\frac{1}{3}(2 + 2\gamma - \beta) = 1.003 \pm .005$ and $\gamma = 1 \pm .001$, in good agreement with Einstein's theory.

However, for various reasons as described in detail in [1], it is worthwhile to consider alternative theories of gravity, the popular ones being the Brans-Dicke (BD) theory with a constant parameter $\omega_{bd} > 0$ [2] and the string theory. A common feature among these theories is the presence of a scalar field ϕ, often called dilaton. The acceptable static spherically symmetric solutions for a charge neutral point star in BD and string theory, including only the graviton and the dilaton field, are all found to give $\beta = \gamma = 1$. There are more general solutions [3] giving $\beta = 1$ and $\gamma = 1 + b$, but they always have naked curvature singularities proportional to b^2 [4] and, hence, are unacceptable. Therefore these theories cannot predict non trivial values for β and/or γ for a charge neutral point star without introducing naked singularities.

In this letter, we look for a graviton-dilaton theory which can predict non trivial values for β and/or γ for a charge neutral point star without introducing naked singularities. The most general graviton-dilaton theory is the generalised BD theory which, with the dilaton potential set to zero, contains one arbitrary function $\psi(\phi)$ [5, 1]. We study static spherically symmetric solutions in this theory and require that the PPN parameters β and/or γ be non trivial for a charge neutral point star, and that there be no naked singularities. See footnote 6 below. These requirements impose certain constraints on ψ which, to the best of our knowledge, are not all satisfied in any of the existing graviton-dilaton theories, including the recent string theoretic model of [6].

These constraints on ψ lead to generic and model independent novel results. For a charge neutral point star, the gravitational force becomes repulsive at distances of the order of, but greater than, the Schwarzschild radius r_0. As a consequence, test particles with non zero rest mass cannot reach r_0. By construction, there is no naked singularity, but now there is also no horizon

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for $r > r_0$. These results, as will be discussed later, suggest that black holes are unlikely to form in a stellar collapse in this theory. Such non formation of black holes, if established rigorously, will obviate the vexing problems of black hole singularities and information loss due to Hawking radiation. We do not understand at present the photon propagation for $r \leq r_0$ and, hence, are unable to determine the space time structure completely. But, this may not be an issue if physical stars do not collapse to point objects. Our theory also has novel, model independent cosmological features which are presented in a companion letter [7].

It is interesting to note that partially similar phenomena also appear in two entirely different models [8, 9]. In [8], a Higgs potential for a complex BD type scalar results in a repulsive gravity. In the 'non symmetric gravitational theory' of [9] the gravitational force vanishes at $r = 0$ and the horizon is absent, leading its authors to conclude that there are no black holes. But this conclusion is debatable [10]. Also, this theory predicts that space is anisotropic and birefringent [11].

Consider now the most general action for the graviton and dilaton, including the matter which determine the space time structure, and the world line actions for the test particles [2, 12, 5] - in our notation, the signature of the metric is $(-+++)$ and $\bar{R}_{\mu\nu\lambda\tau} = \frac{1}{2}\frac{\partial^2 \bar{g}_{\mu\lambda}}{\partial x^\nu \partial x^\tau} + \cdots$:

$$
\begin{aligned}
S &= \int d^4x \sqrt{-\bar{g}}\, e^{\bar{\psi}} \left(\bar{R} - \bar{\omega}(\bar{\nabla}\bar{\phi})^2 + \bar{V} \right) + S_M(\{\chi\}, \{\bar{\sigma}\}, \bar{g}_{\mu\nu}) \\
&\quad + \sum_i m_i \int (-e^{\bar{\sigma}_i}\bar{g}_{\mu\nu}dx_i^\mu dx_i^\nu)^{\frac{1}{2}} \ ,
\end{aligned}
\tag{1}
$$

where $\bar{\psi}$, $\bar{\omega}$, $\bar{V}$, and $\bar{\sigma}_i$ are functions of the scalar $\bar{\phi}$, related to the dilaton (see below), S_M is the matter action with $\{\chi\}$ and $\{\bar{\sigma}\}$ denoting symbolically matter and their dilatonic couplings, m_i's are constants, and the sum is over different types of test particles at least some of which have, by assumption, non zero rest mass, i.e. $m_i \neq 0$ at least for some i.[1] We also assume that matter constituents have non zero rest mass. Due to the dilatonic couplings, the test particles feel dilatonic forces and, hence, will not fall freely along the geodesics of $\bar{g}_{\mu\nu}$ [12, 5].

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[1]In the following, unless otherwise stated, test particles refer to those only with non zero rest mass.

In the following, we will set the dilaton potential $\bar{V} = 0$.[2] When $\{\bar{\sigma}\}$ and $\bar{\sigma}_i$'s are different, it amounts to introducing a composition dependent 'fifth force' (see [12, 5, 1]). Hence, we take $\{\bar{\sigma}\} = \bar{\sigma}_i = \bar{\sigma}$ for all i, thus coupling dilaton universally to all matter and test particles.

Now, the action in (1) appears to have three arbitrary functions $\bar{\psi}$, $\bar{\omega}$ and $\bar{\sigma}$. However, first by a $\bar{\phi}$-dependent conformal transformation of $\bar{g}_{\mu\nu}$, and then by a scalar redefinition of $\bar{\phi}$, two of these arbitrary functions can be gotten rid of. The action (1) can then be written as:

$$
\begin{aligned}
S \;=\; & \int d^4x \sqrt{-g}\left(R + \frac{1}{2}(\nabla\phi)^2\right) + S_M(\{\chi\}, e^{-\psi} g_{\mu\nu}) \\
& + \sum_i m_i \int \left(-e^{-\psi} g_{\mu\nu} dx_i^\mu dx_i^\nu\right)^{\frac{1}{2}} \; ,
\end{aligned}
\tag{2}
$$

where the redefined scalar is referred to as the dilaton ϕ and the arbitrary function $\psi(\phi)$ charcterises our graviton-dilaton theory. ψ cannot be gotten rid of by any further field redefinitions (except when *all* matter constituents and test particles have zero rest mass which, by assumption, is not the case here). Note that in (2) the curvature scalar R appears canonically. For this reason, $g_{\mu\nu}$ is often referred to as 'Einstein metric'. However, matter and test particles couple to dilaton now and feel both the gravitational and the dilatonic forces. Hence, they do not fall freely along the geodesics of $g_{\mu\nu}$.

The action in (2) can be written equivalently in terms of the metric

$$
\hat{g}_{\mu\nu} = e^{-\psi} g_{\mu\nu} \; .
\tag{3}
$$

It then becomes

$$
\begin{aligned}
S \;=\; & \int d^4x \sqrt{-\hat{g}}\, e^{\psi} \left(\hat{R} - \omega(\hat{\nabla}\phi)^2\right) + S_M(\{\chi\}, \hat{g}_{\mu\nu}) \\
& + \sum_i m_i \int \left(-\hat{g}_{\mu\nu} dx_i^\mu dx_i^\nu\right)^{\frac{1}{2}} \; ,
\end{aligned}
\tag{4}
$$

where $\omega = \frac{1}{2}(3\psi_{(1)}^2 - 1)$. Here $\psi_{(n)} \equiv \frac{d^n\psi}{d\phi^n}$, the nth derivative of ψ with respect to ϕ. In (4) and in the following, hats denote quantities involving $\hat{g}_{\mu\nu}$. Note that in (4) the curvature scalar $\hat{R}$ does not appear canonically. However,

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[2]Static spherically symmetric graviton-dilaton solutions, with $\bar{\psi} = \bar{\phi}$ and $\bar{\omega} = 1$, have been studied in [4, 13] for different choices of $\bar{V}$.

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matter and test particles now couple to the metric only canonically and, hence, fall freely along the geodesics of $\hat{g}_{\mu\nu}$. For this reason, we refer to $\hat{g}_{\mu\nu}$ as physical metric: since matter and test particles follow its geodesics, the PPN parameters or the singularities related to $\hat{g}_{\mu\nu}$ are the physically relevant quantities. This is the original Dicke's framework [2, 12] (perhaps it is more appropriate to refer to $\hat{g}_{\mu\nu}$ as Dicke metric).

It should be noted, however, that both forms of action given in (2) and (4) are equivalent. It is clear that, with $\hat{g}_{\mu\nu}$, the physical quantities are directly obtained from the metric whereas, with $g_{\mu\nu}$, the dilatonic force must also be taken into account. On the other hand, it turns out that equations of motion are easier to solve using Einstein metric. In fact, in the following we will utilise both of these aspects.

Thus, our theory is specified by the action for graviton, dilaton, matter, and test particles given in (2) or, equivalently, in (4). It is characterised by one arbitrary function $\psi(\phi)$. Note that setting $\psi = \phi \, (3 + 2\omega_{bd})^{-\frac{1}{2}}$ in (4) one gets Brans-Dicke theory; and, setting $\psi = \phi$ in (4) one gets the graviton-dilaton part of the low energy string theory[3].

The equations of motion obtained from (2) are

$$
\begin{aligned}
2R_{\mu\nu} &= -\nabla_\mu \phi \nabla_\nu \phi - e^{-\psi}\left(T_{\mu\nu} - \frac{g_{\mu\nu}}{2}e^{-\psi}T\right) \\
2\nabla^2\phi &= \psi_{(1)}e^{-2\psi}T \,,
\end{aligned}
\tag{5}
$$

where

$$
T_{\mu\nu} \equiv \frac{2}{\sqrt{-\hat{g}}}\frac{\delta S_M}{\delta \hat{g}^{\mu\nu}} \quad \text{and} \quad T \equiv \hat{g}_{\mu\nu}T^{\mu\nu}
$$

are physical energy-momentum tensor of the matter and its trace respectively.

We now consider static spherically symmetric vacuum solutions. Hence, we set $T_{\mu\nu} = T = 0$ in (5), and first solve for ϕ and $g_{\mu\nu}$. We then obtain the physical metric $\hat{g}_{\mu\nu}$ using (3) and the physical curvature scalar $\hat{R}$ using

$$
\hat{R} = e^{\psi}(R + \frac{3}{2}(\nabla\psi)^2 - 3\nabla^2\psi) \,,
\tag{6}
$$

and study the physical PPN parameters and the singularities.

[3]modulo the choice of dilatonic couplings. In the string theory literature, the action in (2), but with $\psi = 0$, and that in (4), but with $\psi = \phi$, have both been used often.

With $\phi = \phi(r)$, the equations of motion in the gauge $ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + h^2(r)d\Omega^2$, where $d\Omega^2$ is the line element on an unit sphere, become

$$\frac{(fh^2)''}{2} - 1 = (f'h^2)' = (\phi'fh^2)' = 0$$

$$4h'' + h\phi'^2 = 0 \tag{7}$$

where $'$ denotes r-derivatives. The most general solution to (7) is [3]:

$$f = Z^a , \quad h^2 = r^2 Z^{1-a} , \quad e^{\phi-\phi_0} = Z^b \tag{8}$$

where $Z \equiv (1 - \frac{r_0}{r})$ and ϕ_0, r_0, a, and b are constants with $a^2 + b^2 = 1$. The constant r_0 is the Schwarzschild radius and is proportional to the physical mass M of the point star and, hence, is positive. The constant b can be positive or negative, but we take the constant a to be positive so that, if $b = 0$ then $a = 1$ and, with $r_0 = 2M$, one gets the standard Schwarzschild solution. Note that Z remains positive and non vanishing for $r > r_0$.

The physical metric $\hat{g}_{\mu\nu}$, using (3), is given by

$$d\hat{s}^2 = -e^{-\psi}f dt^2 + \frac{e^{-\psi}}{f}dr^2 + e^{-\psi}h^2 d\Omega^2 . \tag{9}$$

Rewriting it in the form $d\hat{s}^2 = -\hat{f}(\rho)dt^2 + \hat{F}(\rho)(d\rho^2 + \rho^2 d\Omega^2)$, where $r = \rho(1 + \frac{r_0}{4\rho})^2$, and expanding $\hat{f}$ and $\hat{F}$ in the limit $\rho \to \infty$, one obtains the physical mass M and the PPN parameters β and γ, defined as follows [1]:

$$\hat{f} = 1 - \frac{2M}{\rho} + \frac{2\beta M^2}{\rho^2} + \cdots , \quad \hat{F} = 1 + \frac{2\gamma M}{\rho} + \cdots .$$

After a straightforward algebra we find, using (8), that

$$2M = (a - b\psi_{(1)}(\phi_0)) r_0$$

$$\beta = 1 - \frac{b^2 r_0^2}{4M^2}\psi_{(2)}(\phi_0)$$

$$\gamma = 1 + \frac{br_0}{M}\psi_{(1)}(\phi_0) , \tag{10}$$

where $\phi_0 = \phi|_{r\to\infty}$, and that the physical curvature scalar

$$\hat{R} = \frac{b^2 r_0^2}{2r^4}(3\psi_{(1)}^2 - 6\psi_{(2)} - 1)e^{\psi}Z^{a-2} . \tag{11}$$

As mentioned above, $b = 0$ gives the standard Schwarzschild solution. Taking $\psi = \phi$, which corresponds to the low energy string theory, we get

$$2M = (a - b)r_0 \ , \ \ \beta = 1 \ , \ \ \gamma = 1 + \frac{br_0}{M} \ .$$

Let $b \neq 0$ so that γ is non trivial. Then, $a - b > 0$ since the physical mass M must be positive. Therefore, the tt-component $\hat{g}_{tt} = -Z^{a-b}$ vanishes at r_0. However, the physical curvature scalar $\hat{R} = b^2 r_0^2 r^{-4} Z^{a+b-2}$ diverges at r_0 since $a + b - 2 < 2(a - 1) < 0$.[4] This singularity will be present for a charge neutral point star if $b \neq 0$; or equivalently, if the PPN parameter $\gamma \neq 1$. Also, as shown in [4], this singularity is naked and, hence, physically unacceptable. Similar situation arises in Brans-Dicke theory also. Thus these theories cannot predict non trivial β or γ for a charge neutral point star, without implying a naked singularity.

We would like to see if it is possible to find a graviton-dilaton theory which does not have this problem. That is, to find a function ψ such that one gets a non trivial value for β and/or γ for a charge neutral point star, without any naked singularities. We take $b \neq 0$ and, hence, $a = \sqrt{1 - b^2} < 1$ from now on. The physical mass M will be positive, and β and/or γ will be non trivial and lie within the experimental bounds if ψ satisfies

$$a - b\psi_{(1)}(\phi_0) > 0 \ , \ \ b^2 \psi_{(2)}(\phi_0) = \delta \ , \ \ b\psi_{(1)}(\phi_0) = \epsilon \ , \tag{12}$$

where δ and ϵ are $< 10^{-3}$, and at least one of them is nonzero.

Consider the singularities. For them to be absent, not only $\hat{R}$ but *all* other curvature invariants must also be finite. As shown in the appendix, any curvature invariant is of the form given in (20). Now, for $r_0 \leq r \leq \infty$, ϕ ranges from ϕ_0 to $\pm\infty$ depending on the sign of b. However, we would like the singularities to be absent in the region $r \geq r_0$, for either sign of b. Hence, we take $-\infty \leq \phi \leq \infty$. Then, from (20) follows a necessary condition for the absence of singularities for $r \geq r_0$:

$$\psi_{(n)}(\phi) \equiv \frac{d^n \psi}{d\phi^n} = (finite) \quad \forall \ n \geq 1 \ \text{and} \ -\infty \leq \phi \leq \infty \ . \tag{13}$$

With equation (13) is satisfied, the necessary and sufficient condition for the absence of singularities at $r = r_0$ is that

$$\lim_{r \to r_0} e^\psi Z^{a-2} = \lim_{\phi \to \pm\infty} e^{\psi - \frac{a-2}{|b|}|\phi|} = (finite) \tag{14}$$

[4]Here we have used $a - b > 0$ and, since $b \neq 0$, $a = \sqrt{1 - b^2} < 1$.

where, using (8), Z is written as a function of ϕ only, valid for either sign of b. Equations (13) and (14) then uniquely imply that

$$\lim_{\phi \to \pm\infty} \psi = -\frac{l}{|b|} \, |\phi| \;, \quad l \geq 2 - a \;,$$

which can be written equivalently as

$$\lim_{\phi \to \pm\infty} \psi = -\lambda \, |\phi| \;, \quad \text{where} \;\; \lambda \geq \frac{2-a}{\sqrt{1-a^2}} \geq \sqrt{3} \tag{15}$$

because $0 < a < 1$. From equations (13) and (15) it now follows that ψ has a finite upperbound, *i.e.* $\psi \leq \psi_{max} < \infty$. Hence, e^{ψ} is finite for $r > r_0$. Since Z is non vanishing for $r > r_0$, it follows from (13) and (20) that the singularities are absent for $r > r_0$ as well.

Therefore, the requirements on ψ are that it satisfy equations (12), (13), and (15). The corresponding graviton-dilaton theory can predict non trivial values of β and/or γ for a charge neutral point star without introducing any singularities for $r \geq r_0$. An example of such a function ψ is given by $\psi = -\lambda\sqrt{(\phi - \phi_1)^2 + c^2}$ where ϕ_1 and c^2 are constants, $\lambda \geq \sqrt{3}$ and the square root is to be taken with positive sign. This function, for a suitable choice of the constants ϕ_1 and c, obviously satisfies equations (12), (13) and (15). Many other examples are easy to obtain.

For any function ψ satisfying only equations (12), (13), and (15), consider now the physical metric $\hat{g}_{\mu\nu}$, in particular its tt-component $\hat{g}_{tt} = -e^{-\psi}Z^a$, $0 < a < 1$. As r decreases from ∞ to r_0, $(-\hat{g}_{tt})$ decreases from 1, reaches a minimum at $r_{min} > r_0$, and then diverges to infinity at r_0, always remaining positive and non vanishing for $r_0 \leq r \leq \infty$. This follows due to the following four reasons. (1) Since the physical mass M is positive,

$$- \hat{g}_{tt} = 1 - \frac{2M}{r} + \mathcal{O}(\frac{1}{r^2}) \; < 1 \;\; \text{as} \;\; r \to \infty \;; \tag{16}$$

(2) $e^{-\psi}$, which is non negative, never vanishes because $\psi \leq \psi_{max} < \infty$; (3) $Z > 0$ for $r > r_0$; and (4)

$$\lim_{r \to r_0}(-\hat{g}_{tt}) = Z^{a-l} \geq \left(1 - \frac{r_0}{r}\right)^{2(a-1)} \to \infty \tag{17}$$

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because $l \geq 2 - a$, and $a < 1$. The exact value of r_{min} is determined from $|b\psi_{(1)}(r_{min})| = a$. It depends on the details of ψ, but is always greater than, and typically of the order of, the Schwarzschild radius r_0.

Qualitatively speaking, the slope of the curve $(-\hat{g}_{tt})$ vs. r indicates the nature of the gravitational force. Thus, for the standard Schwarzschild black hole where $(-\hat{g}_{tt}) = 1 - \frac{2M}{r}$, the force is always attractive. In our case, then, the r-dependence of $(-\hat{g}_{tt})$ described above indicates that the gravitational force is attractive for $r > r_{min}$, vanishes at r_{min}, and becomes repulsive for $r < r_{min}$.

The repulsive force can also be seen by studying the geodesic motion of a radially incoming test particle with non zero rest mass. For a metric given by $ds^2 = -g_0 dt^2 + g_1 dr^2 + g_2 d\Omega^2$, where g_0, g_1, and g_2 are functions of r only, the radial geodesic equation becomes

$$r_{pp} + \frac{g_1' r_p^2}{2g_1} + \frac{g_0'}{2g_1 g_0^2} = 0 , \quad t_p = \frac{1}{g_0} , \tag{18}$$

where $(\ )' \equiv \frac{d(\ )}{dr}$ and $(\ )_p \equiv \frac{d(\ )}{dp}$. Equation (18) can be integrated twice to get

$$\int dt = \int dr \sqrt{\frac{g_1}{g_0(1 + Eg_0)}} \tag{19}$$

where $E = -1 + v^2$, corresponding to releasing the test particle at $r = \infty$ with an inward velocity v (in units where velocity of light $= 1$). Since the test particle has non zero rest mass, its velocity $v < 1$ and, hence, $E < 0$. In our case $g_0 = -\hat{g}_{tt}$ which, as shown above, diverges to ∞ at r_0. Therefore, the denominator in (19) vanishes at some r_t, where $1 + Eg_0(r_t) = 0$ and $r_0 < r_t < r_{min}$, indicating that r_t is the turning point. Analysis of equation (18) then shows that the test particle travels outwards after reaching r_t. It is clear that such a turning point exists irrespective of the value of $v \ (< 1)$ or, equivalently, the initial energy of the test particle. This shows that test particles feel a repulsive gravitational force near r_0.[5] In Einstein frame, where the action is given by (2), this repulsion can be thought of as arising due to dilatonic force.

The repulsive gravitational force has novel implications for the collapse of a star. Consider a non rotational collapse and assume, in the following,

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[5]Contrast this with the Schwarzschild black hole where $g_0 = 1 - \frac{r_0}{r} \leq 1$: the factor $1 + Eg_0$ never vanishes and, hence, there is no turning point.

that the star is so massive that its matter pressure is insufficient to prevent gravitational collapse.

In Einstein's theory, such a star collapses completely, becoming a black hole [14] (see also [15]). In [16], it has been shown that if a black hole forms in a stellar collapse in Brans-Dicke theory, then the dilaton will become constant everywhere. Thus, the original dilaton field, if any, of a star will all be radiated away when it collapses and becomes a black hole. But, reference [16] does not address the question of whether or not a star in Brans-Dicke theory *will* collapse to become a black hole. Recently, this question has been answered affirmatively in [17], where the authors show that the star in Brans-Dicke theory indeed collapses and becomes a black hole, radiating away all the dilaton field in the process.[6] This is the standard collapse scenario.

The graviton-dilaton theory described in this paper suggests naturally the following collapse scenario. Let r_* be the radius of the star. The solutions presented here may describe the gravitational field outside r_*, while the fields inside may be obtained by solving the matter coupled equations and imposing appropriate boundary conditions at r_*. At equilibrium, r_* cannot be less than r_{min} since the gravitational force becomes repulsive for $r < r_{min}$, causing the star to expand. During a collapse, when r_* becomes $< r_{min}$, the gravitational repulsion may halt and reverse the collapse, eventually stabilising r_* around r_{min}. Since there is no horizon for $r > r_{min}$ $(> r_0)$, a stellar collapse in this theory may not result in the formation of a black hole, even if the star is supermassive. Note that the repulsive gravitational force is crucial in this scenario.

Is this scenario plausible? We think that the answer is yes, for the following reasons. The standard scenarios in [14]-[16] do not apply to our case. In Einstein frame, the gravitational repulsion can be thought of as arising due to the dilatonic coupling of matter. The matter in [14, 15] has no dilatonic coupling and, hence, no repulsive force will ever arise, making the analyses of [14, 15] inapplicable to our case. An important reason for the numerical simulation of [17] itself is that reference [16] sidesteps the question of whether or not a collapse in Brans-Dicke theory results in a black hole. The same

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[6]Thus, a charge neutral star can have non trivial β and γ. When it collapses and forms a black hole, the dilaton evaporates away producing $\beta = \gamma = 1$. This, then, weakens the original motivation for the construction of the present graviton-dilaton theory. Nevertheless, we wish to continue its study in view of the simplicity of the initial requirements and the consequent generic novel features.

objection and the fact that gravity remains attractive in BD theory make the analysis of [16] inapplicable to our case also.

Consider now the relevant points from the analysis of [17]. The collapse simulation is carried out for $\omega_{bd} \geq 0$, where ω_{bd} is the constant BD parameter. It is carried out for $\omega_{bd} < -2$ also, but this will not be relevant to our purpose. The simulation has not been carried out for $-2 < \omega_{bd} < 0$ due to the subtleties arising from the singular behaviour of the factor $(2\omega_{bd} + 3)^{-1}$ appearing in the source term in the dilaton equation. Also, the gravitational force always remains attractive in BD theory.

In the present theory, the gravitational force becomes repulsive near r_0. The analog of the BD parameter now is $\omega_{bd}(\phi) = -\frac{3}{2} + \frac{1}{2\psi_{(1)}^2}$, a function of dilaton [7]. Near r_0, where the gravitational force becomes repulsive, $-\frac{3}{2} \leq \omega_{bd}(\phi) \leq -\frac{4}{3}$ because $\psi_{(1)}^2 = \lambda^2 \geq 3$ (see equation (15)). These values of $\omega_{bd}(\phi)$ are the ones likely to be relevant to our collapse scenario, and are precisely those that were not investigated in [17]. Hence, this work does not rule out the collapse scenario in the present theory, described above [18].

Consider equations (5). They describe the dynamics of collapse. In the dilaton equation, qualitatively speaking, the metric coupling in $\nabla^2 \phi$ causes the dilaton to evaporate away during the collapse to a black hole, whereas the matter/self-coupling term $\psi_{(1)} e^{-2\psi} T$ acts as a source for ϕ.

Consider these terms. For a metric and the dilaton of the form similar to that given in (8), and for a slow enough collapse, the terms in $\nabla^2 \phi$ diverge near r_0 at most as Z^{-2}, where $Z = 1 - \frac{r_0}{r}$ and r_0 is the Schwarzschild radius, now likely to depend on both r and t. In the source term, T is non divergent. $\psi_{(1)}$ is a constant and is < 1 in BD theory; it is also a constant, but $>> 1$, near r_0 in our theory. The factor $e^{-2\psi}$ diverges near r_0 as Z^{-2k} where, in BD theory, $k = \frac{b}{3+2\omega_{bd}} << 1$ since b is small and ω_{bd} is large. Thus, in BD theory, the gravitational effects on ϕ can be expected to dominate the source effects. This can also be seen in [17], where the dilaton initially grows in magnitude and starts evaporating away just when a horizon is forming (see figures 1-5 in [17]). In the present theory, on the other hand, for any function ψ that satisfies the constraints given in (12), (13), and (15), $k = l \geq (2 - a) > 1$ since $a < 1$. It implies that source effects become dominant now! This makes it plausible that despite the metric induced evaporation, the dilaton may still grow in magnitude. In that case, the gravitational repulsion would set in, and the collapse is likely to be halted as in the scenario proposed above.

Because of these reasons, we think that the standard collapse scenario may not be applicable in the present graviton-dilaton theory, and that the scenario we have proposed may be the right one. The precise dynamics of collapse, however, needs to be understood in full detail, but its analytical study is quite difficult. It can, perhaps, be best understood by numerical simulations only as in [17], and is presently under study.

Consider now test particles with zero rest mass (photons), propagating in the space time given by (8). Strictly outside r_0, the nature of their propagation is similar to that in the Schwarzschild case. For example the redshift, as also the traversal time measured in a laboratory, of a photon travelling from any $r_i > r_0$ to $r_f > r_i$ is finite. Since $\hat{g}_{tt}$ is non vanishing and there are no singularities in this region, one concludes that there is no horizon for $r > r_0$. However, we do not understand at present the photon propagation for $r \leq r_0$. The problem we face is that the dilaton and the metric components become complex when $r < r_0$. Also, the factor $e^{-\psi}$ diverges at $r = r_0$ and, hence, the validity of the conformal transformation (3) appears to be questionable. Consequently, we are unable to determine completely the space time structure.

It may turn out to be the case that in physical situations one need not worry about the region $r \leq r_0$. If the collapse scenario described above holds good, then the radius r_* of any star $> r_0$. The present solutions are then applicable only outside the star, *i.e.* for $r > r_* > r_0$. The solutions inside the star, *i.e.* for $r < r_*$ (which includes the region $r \leq r_0$), will be quite different from the present ones. Then, determining the space time structure may not be an issue. This is unlike in the Schwarzschild case where a black hole can form in physical situations, *i.e.* r_* can be $< r_0$ and, hence, one does need to worry about the region $r \leq r_0$.

To summarise, we have constructed a graviton-dilaton theory by requiring that it can predict non trivial values of the PPN parameters β and/or γ for a charge neutral point star, without any naked singularity. It contains one function $\psi(\phi)$. Our simple requirements constrain ψ to satisfy equations (12), (13), and (15). These constraints lead to generic and model independent novel results as described here. The cosmological features of this theory are presented in a companion letter [7].

Keeping in view the simplicity of the initial requirements, and the resulting novel features that, if established rigorously, may have far reaching consequences, we believe that further study of the present graviton-dilaton

theory will be fruitful.

Acknowledgements:

Appendix

All curvature invariants can be constructed using metric tensor, Riemann tensor, and covariant derivatives which contain ordinary derivatives and Christoffel symbols $\hat{\Gamma}^c_{ab}$. When the metric is diagonal, every term in any curvature invariant can be grouped into factors, each of which is of one of the following forms (no summation over repeated indices): (A) $\sqrt{\hat{g}^{aa}\hat{g}^{bb}\hat{g}^{cc}\hat{g}^{dd}}\hat{R}_{abcd}$, (B) $\sqrt{\hat{g}^{aa}\hat{g}^{bb}\hat{g}_{cc}}\hat{\Gamma}^c_{ab}$, or (C) $\sqrt{\hat{g}^{aa}}\partial_a$.

Taking $\hat{g}_{\mu\nu}$ given in (9), and the solutions for f, h, and ϕ given in (8), the above forms can be calculated explicitly. The calculation is straightforward, and the result is that (A), (B), and (C) can be written, symbolically, as

$$(A) \simeq U e^{\psi} Z^{a-2} , \quad (B) \simeq V(e^{\psi}Z^{a-2})^{\frac{1}{2}} ,$$
$$(C)^n \cdot (A)^p (B)^q \simeq W_{n+2}(e^{\psi}Z^{a-2})^{p+\frac{1}{2}(q+n)} ,$$

where U and V are functions of r, $\psi_{(1)}$, and $\psi_{(2)}$ only, and W_k are functions of r and $\psi_{(l)}$, $1 \leq l \leq k$. As a result of the way various factors are grouped, it turns out that, the explicit r-dependent parts in U, V, and W_k's are finite for all $r \geq r_0$ (in fact, they diverge only at $r = 0$).

Hence, any curvature invariant constructed from m Riemann tensors, n covariant derivatives, and the requisite number of metric tensors will be of the form

$$\tilde{W}(r; \psi_{(1)}, \psi_{(2)}, \cdots, \psi_{(n+2)}) \, (e^{\psi}Z^{a-2})^{m+\frac{n}{2}} \tag{20}$$

where the explicit r-dependent parts of $\tilde{W}$ are finite for all $r \geq r_0$ (in fact, they diverge only at $r = 0$). Note, as an example, that the curvature scalar

given in (11) belongs to type (A), and has the above form with $m = 1$ and $n = 0$.

References

[1] C. M. Will, Theory and Experiment in Gravitational Physics, revised Edition, Cambridge University Press, 1993.

[2] C. Brans and R. H. Dicke, Phys. Rev. **124** (1961) 925.

[3] H. Buchdahl, Phys. Rev. **115** (1959) 1325; A. I. Janis, D. C. Robinson, and J. Winicour, Phys. Rev. **186** (1969) 1729.

[4] S. Kalyana Rama, hepth/9411076, 9505151.

[5] P. G. Bergmann, Int. Jl. Phys. **1** (1968) 25; R. V. Wagoner, Phys. Rev. **D1** (1970) 3209. See also K. Nordtvedt Jr., Astrophys. Jl. **161** (1970) 1059; J. D. Bekenstein, Phys. Rev. **D15** (1977) 1458; J. D. Bekenstein and A. Meisels, Phys. Rev. **D18** (1978) 4378.

[6] T. Damour and A. M. Polyakov, Nucl. Phys. **B423** (1994) 532.

[7] S. Kalyana Rama, hepth/9506020, to appear in Physics Letters **B**.

[8] T. Padmanabhan, J. Phys. **A**: Math. Gen. **16** (1983), 335;
B. S. Satyaprakash, E. A. Lord, and K. P. Sinha, Phys. Lett. **A105** (1984) 407; B. S. Satyaprakash, P. Goswami, and K. P. Sinha, Phys. Rev. **D33** (1986) 2196.

[9] F. Flam, physics research news in Science **266** (1994) 1945;
N. J. Cornish and J. W. Moffat, Phys. Lett. **B336** (1994) 337;
J. Math. Phys. **35** (1994) 6628.

[10] L. M. Burko and A. Ori, Phys. Rev. Lett. **75** (1995) 2455.

[11] M. D. Gabriel *et al*, Phys. Rev. **D43** (1991) 308, 2465.

[12] R. H. Dicke, Phys. Rev. **125** (1962) 2163.

[13] S. Mignemi and D. L. Wiltshire, Class. Quant. Grav. **6** (1989) 987, Phys. Rev. **D46** (1992) 1475; D. L. Wiltshire, Phys. Rev. **D44** (1991) 1100; R. Gregory and J. A. Harvey, Phys. Rev. **D47** (1993) 2411; J. H. Horne and G. Horowitz, Nucl. Phys. **B399** (1993) 169; S. J. Poletti and D. L. Wiltshire, Phys. Rev. **D50** (1994) 7260; C. P. Burgess, R. C. Myers, and F. Quevedo, Nucl. Phys. **B442** (1995) 75; S. Kalyana Rama, Mod. Phys. Lett. **A10** (1995) 789; S. J. Poletti, J. Twamley, and D. L. Wiltshire, hepth/9502054.

[14] S. W. Hawking, Comm. Math. Phys. **25** (1972) 152, and references therein.

[15] D. Christodoulou, Comm. Math. Phys. **105** (1986) 337, **106** (1986) 587, **109** (1987) 613.

[16] S. W. Hawking, Comm. Math. Phys. **25** (1972) 167.

[17] M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. **D51** (1995) 4208, 4236.

[18] M. A. Sheel, and S. A. Teukolsky, private communication.